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# FINE-SCALE MIXING IN A GAS-FLUIDIZED BED OF FINE PARTICLES

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The coefficients of diffusion of the particles and gas in the dense phase of a fluidized bed and the mean squares of the components of their pulsation velocity are analyzed with direct particle collisions neglected.

The particles and the fluidizing medium in a homogeneous fluidized bed or the dense phase of an inhomogeneous bed undergo intense chaotic ("pseudoturbulent") motions, the presence of which leads to the fact that the effective values of the transfer coefficients in the dense phase usually far exceed the values of the corresponding coefficients for the homogeneous materials of the phases. Although in actual inhomogeneous systems the role of such fine-scale mixing can be insignificant within the limits of the bed as a whole compared with the role of the mixing due to the circulation of the phases and bubbling (see [1], for example), it is important precisely for the determination of such quantities as the average time the particles remain near the surface of bodies submerged in the bed and in contact with elements of the dense phase ("packets") and the intensity of particle exchange between the surface zone and the cores of packets.

In particular, the latter quantities determine the intensity of heat exchange between the bed and a submerged surface, and knowledge of their dependence on the operating and physical parameters is entirely necessary for the generalization and further development of the existing particular models of external heat exchange in a fluidized bed [2]. And the

kinetics of some of the more specific processes carried out in a fluidized bed, such as the process of removal of the process binder from ceramic articles obtained by the hot-casting method in a fluidized bed of porous particles through capillary impregnation of particles during their contact with the surface of an article, depends on the same quantities [3, 4]. Similarly, pseudoturbulent pulsations of the fluidizing medium must affect the values of the effective coefficients of dispersion of an impurity in the dense phase, which seriously influence the mass exchange between the bed and bodies bathed by it [5-7] and the intensity of gas exchange between the dense phase and rising bubbles [1, 5], determining the productivity of catalytic reactors.

The average characteristics of the pseudoturbulent motion in a fluidized bed can be found by treating the pulsations of the velocities of the phases, the porosity, and the pressure as steady random functions of time and the coordinates and by using the well-known methods of the correlation theory of steady random processes [8] and the general theory of pseusoturbulence, a survey of which is presented in [9]. The concrete calculations in [9] were made in application to a bed of fine particles fluidized by a liquid, and semiempirical representations were used for the equations of motion of the phases and the force of interphase interaction in which a term proportional to the pressure gradient was incorporated without foundation (see [10], for example). More rigorous results involving the description of the motion of the liquid phase of the disperse medium and the interaction between phases obtained in [11] were used in that report.

Below we consider only beds fluidized by a gas, when it is permissible to neglect the inertia of the gas and the viscous stresses in it in a first approximation. We assume that the particles are fine enough so that the influence of their direct collisions on the exchange of momentum and energy can be neglected, assuming that their interaction takes place predominantly by means of the random fields of velocity and pressure in the gas. The same assumption was adopted in [9]; the opposite situation, when collisions dominate in the exchange of momentum and energy, was investigated in [12].

On the basis of [11], under the indicated assumptions, the equations of conservation of momentum and mass of the gas, which we assume to be incompressible, the equations of particle motion in the dense phase can be written in the form

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial r}(\varepsilon v) = 0, \quad 0 = -\frac{\partial p}{\partial r} - f,$$

$$d_1 \rho \left( \frac{\partial}{\partial t} + w \frac{\partial}{\partial r} \right) w = f - d_1 \rho g, \quad \rho = 1 - \varepsilon,$$

(1)

with a term containing the pressure gradient not having to be introduced in the force f of interaction of the phases per unit volume of the mixture. Using Ergun's well-known two-term equation [13], approximately correct when $\rho \geqslant 0.3$, we write

$$f = d_0 \rho (\beta_1 K_1 + \beta_2 K_2 u) u, \quad u = v - w,$$

(2)

where we introduce the quantities

$$\beta_1 = \frac{75}{2} \frac{v_0}{a^2}, \quad \beta_2 = \frac{1,75}{2a}, \quad K_1(\rho) = \frac{\rho}{\varepsilon^2}, \quad K_2(\rho) = \frac{1}{\varepsilon}.$$

(3)

In [9] a term containing the pressure gradient was introduced into the expression for f, which led to the fact that the functions $K_1$ and $K_2$ in [9] analogous to those in (3) were $\varepsilon^{-1}$ times smaller than the latter.

Each of the unknowns in (1) and (2) can be represented in the form of the sum of its average value and of the pulsation about the average in accordance with the equation $\varphi = \langle \varphi \rangle + \varphi'$. Using the local convective coordinate system x connected with the average particle motion, and linearizing Eqs. (1) and (2) in this system, we obtain a system of linear equations for the pulsations. Representing all the pulsations in the form of Fourier−Stieltjes integrals [8, 9]

$$\phi' = \int \exp\left[i\,(\omega t + \mathbf{kx})\right] dZ_\varphi, \tag{4}$$

we have the following system of equations for the spectral measures:

$$(\omega + \langle \mathbf{u} \rangle \mathbf{k})\,dZ_\rho - \langle \varepsilon \rangle \mathbf{k}dZ_v = 0, \quad -i\mathbf{k}dZ_p - dZ_f = 0,$$

$$id_1\langle \rho \rangle \omega dZ_w = dZ_f, \quad \langle \mathbf{u} \rangle = \langle \mathbf{v} \rangle - \langle \mathbf{w} \rangle, \quad \langle \rho \rangle = 1 - \langle \varepsilon \rangle, \tag{5}$$

and for the spectral measure $dZ_f$ we obtain from (2)

$$dZ_f = d_0\langle \rho \rangle \left[(\beta_1 K_1 + \beta_2 K_2 \langle u \rangle)\,dZ_u + \beta_2 K_2(\langle \mathbf{u} \rangle \, dZ_u)\langle \mathbf{u} \rangle / \langle u \rangle \right.$$

$$\left. + (\beta_1 K_1' + \beta_2 K_2' \langle u \rangle)\langle \mathbf{u} \rangle\,dZ_\rho\right], \quad dZ_u = dZ_v - dZ_w, \tag{6}$$

where the prime to $K_j$ (j = 1, 2) denotes differentiation with respect to $\langle \rho \rangle$.

The average quantities (5) and (6) satisfy the equations obtained from (1) and (2) after averaging, as well as the equation of conservation of mass of the disperse phase; henceforth they are treated as known. In general these quantities describe the local state of a bed or its dense phase occuring when the bed flows over obstacles, when bubbles move in it, etc. In a particular case they can also describe an idealized homogeneous state whose characteristics do not depend on the coordinates. In this case the local coordinate system practically coincides with the laboratory system, and the average relative gas velocity <u> = U = U$_f$/<ε> is directed upward and can be calculated from the condition of equality of the hydraulic forces acting on the particles to their weight minus the buoyant force (which can be neglected for fluidization by a gas). Below we will use the approximate equation

$$U = \frac{\varkappa g}{\beta_1 K_1 + \beta_2 K_2 U} = \frac{\varkappa g}{\beta_1 K_1}\frac{\langle \rho \rangle}{\langle \rho \rangle + 0.0233\,\mathrm{Re}}, \tag{7}$$

$$\mathrm{Re} = \frac{aU_f}{\nu_0}, \quad \varkappa = \frac{d_1}{d_0},$$

corresponding to Ergun's equation [13], in which we neglect the difference between the resistance of a fluidized bed and that of a stationary bed of the same particles due to the pseudo-turbulent motions (see [9], as well as [14]).

Equations (5) and (6) allow us to express all the spectral measures through the measure $dZ_\rho$ of pulsation of the volumetric concentration of particles. Defining the spectral density of the random functions $\varphi'$ and $\psi'$ in the standard way [8],

$$\Psi_{\varphi,\psi}(\omega,\ \mathbf{k}) = \langle dZ_\varphi dZ_\psi^* \rangle / d\omega d\mathbf{k}, \tag{8}$$

we express all the spectral densities of interest through the spectral density of the quantity $\rho'$, for which we have [15]

$$\Psi_{\rho,\rho}(\omega,\ \mathbf{k}) = \frac{\mathbf{kDk}}{\pi}\frac{\Phi_{\rho,\rho}(k)}{\omega_2 + (\mathbf{kDk} - T_0\omega^2)^2}, \quad T_0 = \frac{\mathrm{tr}\,\mathbf{D}}{\langle w'^2 \rangle},$$

$$\Phi_{\rho,\rho}(k) = \begin{cases} \Phi, & k < k_0, \\ 0, & k > k_0 \end{cases}, \quad \Phi = \frac{3}{4\pi}\frac{\langle \rho \rangle^2}{k_0^3}\left(1 - \frac{\langle \rho \rangle}{\rho_m}\right), \tag{9}$$

$$k_0 = \frac{1}{a}\left(\frac{9\pi\langle \rho \rangle}{2}\right)^{1/3}.$$

This expression allows us to close the theory being developed. The various spatial and temporal correlation functions can then be obtained by the standard methods. In such a case [8]

$$\langle \varphi'(t,\ \mathbf{x})\psi'(t + \tau,\ \mathbf{x} + \xi) \rangle = \int\int \exp\left[-i\,(\omega\tau + \mathbf{k}\xi)\right]\Psi_{\varphi,\psi}(\omega,\ \mathbf{k})\,d\omega d\mathbf{k}. \tag{10}$$

Below we investigate only the simplest average characteristics of the pseudoturbulent motion with $\tau = 0$ and $\xi = 0$. To simplify the writing we henceforth omit the angle brackets in the designations of the average quantities.

If the $x_1$ axis of the coordinate system is directed along the vector u, then the tensors D and E of the coefficients of pseudoturbulent diffusion of particles and gas are diagonal ones, with $D_2 = D_3$ and $E_2 = E_3$ [9]. For the formal representation of $D_j$ and $E_j$ ($j = 1, 2$) through the corresponding diagonal elements of the Lagrangian correlation tensor of the velocity of the particles or fluid we use the well-developed theory of the dispersion of fluid moles in a turbulent field, according to which the mean square of the displacement of a mole in the j-th direction over a time far exceeding the correlation "lifetime" can be described as the result of a random process of the diffusional type [16-18]. Then for the coefficients of diffusion of particles and gas we have

$$D_j = \int_0^\infty R_j^{(p)}(\tau)\,d\tau, \quad E_j = \int_0^\infty R_j^{(g)}(\tau)\,d\tau. \tag{11}$$

In the coordinate system under consideration the Lagrangian correlation functions for the components of the particle pulsation velocity coincide with the Eulerian functions within the limits of the adopted accuracy; for a gas the analogous circumstance holds in the local coordinate system connected with the average gas motion, i.e., one moving with a velocity u relative to the system under consideration. We can change to such a system if we replace the frequency $\omega$ by the quantity $\omega - \mathbf{uk}$ in all the equations obtained in the coordinate system under consideration. Consequently, we can write

$$R_j^{(p)}(\tau) = \iint \exp(-i\omega\tau)\,\Psi_{wj,\ wj}\,(\omega,\ \mathbf{k})\,d\omega dk, \tag{12}$$

$$R_j^{(g)}(\tau) = \iint \exp(-i\omega\tau)\,\Psi_{vj,\ vj}\,(\omega - \mathbf{uk},\ \mathbf{k})\,d\omega dk,$$

where the integration extends over the range of variation of the frequency $\omega$ of from $-\infty$ to $\infty$ and over all wave space $\mathbf{k}$. Changing the order of integration over $d\tau$ and $d\omega$ in (11) with allowance for (12) and using the definition of the delta function in the form of a Fourier integral, we obtain

$$D_j = \pi \int \Psi_{wj,\ wj}\,(0,\ \mathbf{k})\,dk, \quad E_j = \pi \int \Psi_{vj,\ vj}\,(-\mathbf{uk},\ \mathbf{k})\,dk. \tag{13}$$

Solving the system (5) with $dZ_f$ from (6) in the general case of $\omega \neq 0$, for the vectors of the spectral measures of the pulsation velocities of particles and gas we have

$$dZ_{wj} = \frac{1}{\varepsilon}\left\{\omega + uk_1\left[1 + \varepsilon\frac{\beta_1 K_1' + \beta_2 K_2' u}{\beta_1 K_1 + \beta_2 K_2 u}\right]\right\}\left\{k^2 + i\varkappa\omega\left[\frac{k_1^2}{\beta_1 K_1 + 2\beta_2 K_2 u} + \frac{k^2 - k_1^2}{\beta_1 K_1 + \beta_2 K_2 u}\right]\right\}^{-1} k_j dZ_\rho, \tag{14}$$

$$dZ_{vj} = \{[i\varkappa\omega + \beta_1 K_1 + (1 + \delta_{1j})\beta_2 K_2 u]\,dZ_{wj} - (\beta_1 K_1' + \beta_2 K_2' u)u\delta_{1j}dZ_\rho\}\,[\beta_1 K_1 + (1 + \delta_{1j})\beta_2 K_2 u]^{-1}. \tag{15}$$

From this it is easy to find the spectral densities figuring in (12) and (13). For example, from (14) we obtain

$$\Psi_{wj,\ wj}(\omega,\ \mathbf{k}) = \frac{k_j^2}{\varepsilon^2}\left\{\omega + uk_1\left[1 + \varepsilon\frac{\beta_1 K_1' + \beta_2 K_2' u}{\beta_1 K_1 + \beta_2 K_2 u}\right]\right\}^2 \times$$

$$\times \left\{k^4 + \varkappa^2\omega^2\left[\frac{k_1^2}{\beta_1 K_1 + 2\beta_2 K_2 u} + \frac{k^2 - k_1^2}{\beta_1 K_1 + \beta_2 K_2 u}\right]^2\right\}^{-1} \Psi_{\rho,\rho}(\omega,\ \mathbf{k}). \tag{16}$$

First of all we find the longitudinal and transverse coefficients of particle diffusion. Using Eqs. (9) and (16) with $\omega = 0$ in (13) and calculating the integrals which appear, we find

$$D_j = \frac{3}{2}(1 + \delta_{1j})\left(\frac{\rho}{\varepsilon}\right)^2\left(1 - \frac{\rho}{\rho_m}\right)\left(1 + \varepsilon\frac{\beta_1 K_1' + \beta_2 K_2' u}{\beta_1 K_1 + \beta_2 K_2 u}\right)\frac{\gamma^2 u^2}{k_0^2 D_2}\,I_j(\gamma), \tag{17}$$
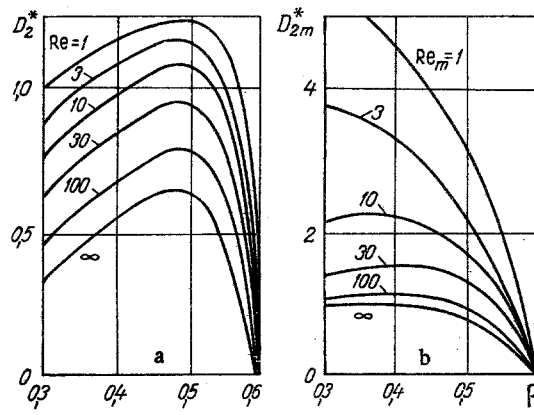
Fig. 1. Theoretical dependences of dimensionless coefficients of transverse particle diffusion on $\rho$: a) (21); b) (23).

where we introduce the notation

$$\frac{1+\gamma^2}{\gamma^2} = \frac{D_1}{D_2}, \quad I_1(\gamma) = \int_0^1 \frac{t^4 dt}{t^2+\gamma^2}, \quad I_2(\gamma) = \int_0^1 \frac{t^2(1-t^2)dt}{t^2+\gamma^2}. \tag{18}$$

From (17) and (18) we obtain the transcendental equation

$$2\gamma^2 I_1(\gamma) = (1+\gamma^2) I_2(\gamma), \tag{19}$$

the only root of which is $\gamma \approx 0.85$. Using this value of $\gamma$ and the definitions in (3), and introducing the quantities

$$u_f = \varepsilon u, \quad D = au_f, \quad \mathrm{Re} = au_f/\nu_0, \tag{20}$$

for the dimensionless coefficients of longitudinal and transverse pseudoturbulent diffusion of particles we obtain from (17)

$$D_1^* = \frac{D_1}{D} \approx 2.4 D_2^*, \tag{21}$$

$$D_2^* = \frac{D_2}{D} \approx 0.32 \frac{\rho^{2/3}}{\varepsilon^2} \left(1 - \frac{\rho}{\rho_m}\right)^{1/2} \frac{1+2\rho+0.0466\,\mathrm{Re}}{\rho+0.0233\,\mathrm{Re}}.$$

Thus, the anisotropy of pseudoturbulent particle diffusion is expressed considerably more weakly for gas fluidization than for fluidization by a liquid [9], with the ratio of the coefficients of longitudinal and transverse diffusion not depending on the average porosity in the given case. The dependences of $D_2^*$ on $\rho = 1 - \varepsilon$ for $\rho_m = 0.6$ and different Re are presented in Fig. 1a; it is seen that these dependences have a single maximum.

In the case when $u_f = U_f$, i.e., Eqs. (21) describe the homogeneous state of a bed or of its dense phase, they are somewhat inconvenient in that the Reynolds number Re, defined in (7) or (20), increases with an increase in $\varepsilon$ and in the fluidization number $N = U_f/U_m$. The quantity Re can take different values for a bed of the same particles in the same gas, while the corresponding values of $D_2^*$ can lie on different curves of Fig. 1a. Therefore, it is convenient to introduce a new Reynolds number $\mathrm{Re}_m = \mathrm{Re}/N$, defined at the moment of the start of fluidization and representing a unique function of the physical parameters of the phases of the bed.

The hydraulic resistance of a granular bed in the fluidized state does not depend on N, and for different $\rho$ and U (or N) it is described by the same analytical expression: Ergun's equation, e.g., which leads to (7). From this we obtain the following connection between
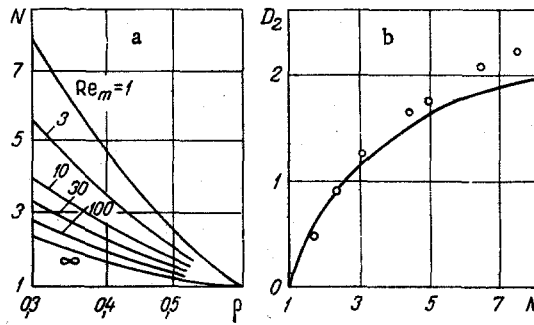
Fig. 2. Dependence of volumetric parti-
cle concentration $\rho$ of the dense phase in
the homogeneous state of a bed on fluidiza-
tion number N (a) and comparison of experi-
mental data of [19] (denoted by points) with
theoretical data for a bed of particles of
quartz sand (b) 0.32 mm in diameter; bed
height 12.5 mm; $Re_m \approx 1$; $U_m \approx 10$ cm/sec.
$D_2$, cm$^2$/sec.

the values of the fluidization number and the average volumetric particle concentration with
$Re_m = $ const:

$$N = \left( \frac{\varepsilon}{\varepsilon_m} \right)^3 \frac{\rho_m + 0.0233\,Re_m}{\rho + 0.0233 N\,Re_m} \ , \ Re_m = \frac{aU_m}{v_0} \ . \tag{22}$$

This dependence of N on $\rho$ is shown in Fig 2a.

In place of (21) we can write

$$D_{1m}^* = D_1/D_m \approx 2.4 D_{2m}^*, \ D_m = aU_m, \tag{23}$$

$$D_{2m}^* = \frac{D_2}{D_m} \approx 0.32\, \frac{\rho^{2/3}}{\varepsilon^2} \left( 1 - \frac{\rho}{\rho_m} \right)^{1/2} \frac{1 + 2\rho + 0.0466 N\,Re_m}{\rho + 0.0233 N\,Re_m} N,$$

where $\rho$ and N are connected by Eq. (22). Equations (23) determine the dependences of the
new dimensionless coefficients of particle diffusion on $\rho$ (or N) for a fixed $Re_m$, i.e., for
a bed of the given particles. Such dependences are plotted in Fig. 1b.

A comparison between the theoretical results obtained and test data is hindered by the
fact that with fluidization by gases we are usually dealing with inhomogeneous beds in which
the average porosity of the dense phase is unknown. However, there are experiments on the
determination of the coefficient of transverse particle diffusion in relatively shallow,
finely dispersed beds, in which an inhomogeneous state is simply unable to become estab-
lished, so that the volumetric fraction of bubbles is small, the bubbles themselves are
small, and the transverse mixing caused by them does not play a significant role. The re-
sults of tests of this type carried out in [19] on beds of quartz sand particles are shown
in Fig. 2b; the theoretical curves are constructed on the basis of Eqs. (22) and (23); the
corresponding values of $\rho$ (or of the porosity $\varepsilon$) are easily found from the curves of Fig. 2a.
The agreement between the theoretical and experimental data appears satisfactory; a certain
systematic difference between them is evidently connected with ignoring the influence of
bubbles.

The determination of the mean squares of the components of the pulsation velocity of
particles in accordance with Eq. (10) is associated with very cumbersome and laborious cal-
culations. To simplify them we note that the characteristic scale $\omega_1$ of variation of the
spectral density in (9), treated as a function of $\omega$, has the order $u/a$, whereas the analo-
gous scale $\omega_2$ of the denominator in (16) has the order of the quantity $\varkappa^{-1}(\beta_1 K_1 + B_2 K_2 u)$.
Using (3) and the definition of Re in (20), for fluidization by a gas we obtain

$$\frac{\omega_2}{\omega_1} \approx \frac{75}{2} \frac{\rho}{\varepsilon} \frac{1 + 0.0233\,\mathrm{Re}/\rho}{\varkappa\,\mathrm{Re}} \sim 5 \cdot 10^{-2} \frac{1 + 0.0233\,\mathrm{Re}/\rho}{\mathrm{Re}}. \tag{24}$$

Obviously, $\omega_2/\omega_1 \ll 1$ if $\mathrm{Re} \gtrsim 0.5$, i.e., for almost all the situations of interest.

Using the estimate (24) in the integration of the quantity (16) over $d\omega$ and neglecting terms of order $\omega_2/\omega_1$, we find

$$\Phi_{wj,\,wj}(\mathbf{k}) = \int_{-\infty}^{\infty} \Psi_{wj,\,wj}(\omega,\,\mathbf{k})\,d\omega \approx \frac{1}{\rho\varepsilon^2} \frac{(1 + 2\rho + 0.0466\,\mathrm{Re})^2}{\rho + 0.0233\,\mathrm{Re}} \times \tag{25}$$

$$\times \frac{\beta_1 K_1 u^2}{\varkappa} \frac{k_i^2\,k_j^2}{k^4} \left[ \frac{k^2 - k_1^2}{k^2} + \frac{\rho + 0.0233\,\mathrm{Re}}{\rho + 0.0466\,\mathrm{Re}} \frac{k_1^2}{k_2^2} \right]^{-1} \frac{\Phi_{\rho,\rho}(k)}{k\,Dk}.$$

Then integrating over $dk$ and using the expressions in (3), (9), and (21), after the calculations we have

$$\langle w_j'^2 \rangle \approx 30\,(1 + \delta_{1j})\frac{\rho^{2/3}}{\varepsilon^4} \left(1 - \frac{\rho}{\rho_m}\right)^{1/2} \frac{1 + 2\rho + 0.0466\,\mathrm{Re}}{\varkappa\,\mathrm{Re}} J_j(\gamma,\,\alpha)\,u_j^2, \tag{26}$$

where

$$\alpha = 0.0233\,\mathrm{Re}\,(\rho + 0.0466\,\mathrm{Re})^{-1},\quad \gamma = 0.85, \tag{27}$$

$$J_1(\gamma,\,\alpha) = \int_0^1 \frac{t^4\,dt}{(t^2/\gamma^2 + 1)(1 - \alpha t^2)},\quad J_2(\gamma,\,\alpha) = \int_0^1 \frac{t^2(1 - t^2)\,dt}{(t^2/\gamma^2 + 1)(1 - \alpha t^2)}.$$

The quantity $\langle w_1'^2 \rangle / \langle w_2'^2 \rangle$ as a function of $\rho$ and Re is shown in Fig. 3, from which it is seen that it depends slightly on its arguments. The curves of Fig. 3 indicate that the anisotropy is far more weakly expressed for fluidization by a gas than for fluidization by a liquid [9]. The dependences of the dimensionless quantity $w_2^{*2} = \langle w_2'^2 \rangle / u_f^2$ on $\rho$ for different Re in application to the fluidization of aluminum oxide particles ($\varkappa = 1.85 \cdot 10^3$) by air are presented in Fig. 4a. The deficiencies of these curves are that, first, they are constructed with Re = const, while Re varies with the variation of the fluidization conditions, and second, they depend on the quantity $\varkappa$, which varies with variation of the physical parameters. In the case when we are dealing with pulsations in a homogeneous bed, these deficiencies can be removed by introducing the criterion $\mathrm{Re}_m$ and the dependence of N on $\rho$ in accordance with (22) and the curves in Fig. 2a. In this case Eq. (26) is converted to the form

$$\langle w_j'^2 \rangle \approx 0.8\,(1 + \delta_{1j})\frac{\rho^{2/3}}{\varepsilon} \left(1 - \frac{\rho}{\rho_m}\right)^{1/2} \frac{1 + 2\rho + 0.0466\,N\,\mathrm{Re}_m}{\rho + 0.0233\,N\,\mathrm{Re}_m} J_j(\gamma,\,\alpha)\,ag, \tag{28}$$

where $\alpha$ and $J_j$ are defined as before by Eqs. (27) with Re = $N\mathrm{Re}_m$. The dependence of the dimensionless quantity $w_{2m}^{*2} = \langle w_2'^2 \rangle / ag$ on $\rho$ for different $\mathrm{Re}_m$ is shown in Fig. 4b. Thus, the dimensional quantity characterizing the intensity of pseudoturbulent particle pulsations is essentially the quantity $ag$.

The results obtained also allow one to determine the "mixing lengths" for particle motion in different directions. Defining the coefficients of diffusion as $D_j = \langle w_j'^2 \rangle^{1/2} l_j$, we obtain $l_j = (D_j^*/w_j^*)a$. Fully, analogously, using Eq. (15) for $dZ_v$ instead of (14), one can investigate the quantities $E_j$ and $\langle v_j'^2 \rangle$: their determination comes down to the calculation of integrals which follow from (10) and (13). Such calculation is very cumbersome; as an example here we obtain only an approximate equation for the coefficient of transverse gas diffusion.

In the approximation of $\omega_2 \ll \omega_1$ (see (24)), from (15) and (16) we obtain

$$\Psi_{v2,\,v2}(-\mathbf{u}\mathbf{k},\,\mathbf{k}) \approx \left(\frac{u}{\varepsilon}\right)^2 \left(\frac{1 + \rho + 0.0233\,\mathrm{Re}}{\rho + 0.0233\,\mathrm{Re}}\right)^2 \frac{k_1^2\,k_2^2}{(k^2 - \alpha k_1^2)^2} \Psi_{\rho,\rho}(-\mathbf{u}\mathbf{k},\,\mathbf{k}). \tag{29}$$
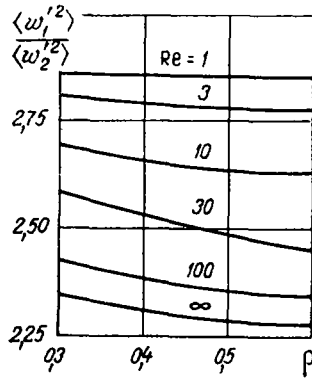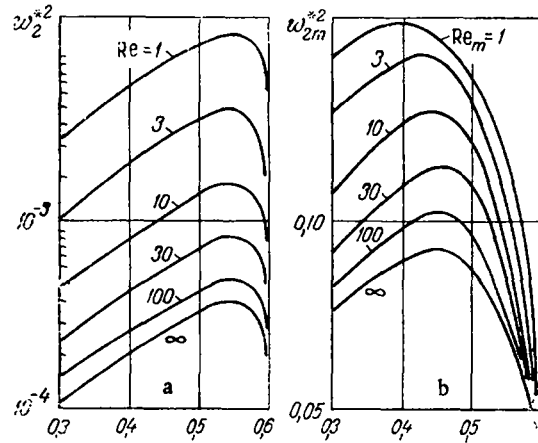
Fig. 3                           Fig. 4

Fig. 3. The ratio $<w_1'^2>$ / $<w_2'^2>$ as a function of $\rho$ and Re.

Fig. 4. Dependences on $\rho$ of the dimensionless quantities $w_2^{*2}$ for a bed of aluminum oxide particles with $\varkappa = 1.85 \cdot 10^3$ (a) and $w_{2m}^{*2}$ (b) for different Re and $Re_m$.

It is easy to show that in calculating the integral (13) of this quantity it is permissible to replace the denominator in Eq. (9) by $\omega^2 = (uk)^2$ with an error not very large. After the calculation we obtain

$$E_2 \approx 0.05 \frac{\rho^{8/3}}{\varepsilon^4} \left(1 - \frac{\rho}{\rho_m}\right)^{3/2} \left(\frac{1 + \rho + 0.0233\,\mathrm{Re}}{\rho + 0.0233\,\mathrm{Re}}\right)^2 \frac{1 + 2\rho + 0.0466\,\mathrm{Re}}{\rho + 0.0233\,\mathrm{Re}} L(\gamma, \alpha) D,$$

(30)

$$L(\gamma, \alpha) = \int_0^1 \frac{(1 - t^2)(t^2/\gamma^2 + 1)}{(1 - \alpha t^2)^2} dt.$$

From this it follows, in particular, that transverse gas diffusion is far weaker than transverse particle diffusion. Such a result can also be obtained for longitudinal diffusion, as well as for the pulsation intensity.

In conclusion, we note that the application of these results in practical estimates and calculations is greatly hindered by the absence of detailed information about the average porosity of the dense phase in the essentially inhomogeneous fluidized beds which are usually used. Without going into detail, we can state that it can now be considered as solidly established that actual beds depart considerably from the demands dictated by the two-phase theory of fluidization, i.e., the indicated porosity departs seriously from $\varepsilon_m$. However, the existing experimental material (see [20], for example) is not sufficient to draw definite conclusions about the character of the dependence of this quantity on the physical and operating parameters of a bed under different conditions. It is clear that the theoretical and experimental determination of this dependence represents one of the central problems of the physical mechanics of fluidization.

NOTATION

$a$, particle radius; $D_j$ and $E_j$, coefficients of pseudoturbulent diffusion of particles and gas; D, quantity introduced in (20); d, density; f, force of interphase interaction; g, acceleration of gravity; $I_j$, $J_j$, L, integrals in (18), (27), and (30); $K_j$, functions introduced in (3); k, wave vector; $k_0$, value of wave number defined in (9); $l_j$, mixing lengths for particles; N, fluidization number; p, pressure; $R_j^{(p)}$, $R_j^{(g)}$, Lagrangian correlation functions for particles and gas; $T_0$, parameter in (9); U, $U_f$, average relative gas velocities, referred to the free through cross section and the total cross section of the apparatus for a homogeneous state of the bed; $U_m$, minimum fluidization velocity; u, $u_f$, average relative

gas velocity and average filtration velocity; $v$, $w$, average velocities of gas and particles; $dZ\varphi$, designation for spectral measures; $\alpha$, parameter introduced in (27); $\beta_j$, coefficients defined in (3); $\gamma$, parameter in (18); $\varepsilon$, porosity; $\varkappa = d_1/d_0$; $\nu$, kinematic viscosity; $\rho$, volumetric particle concentration; $\Psi_{\varphi,\psi}$, $\Phi_{\varphi,\psi}$, designations for spectral densities; $\Phi$, parameter in (9); $\omega$, frequency; $Re$, $Re_m$, Reynolds numbers introduced through velocities $u_f$ (or $U_f$) and $U_m$. Indices: 0 and 1, continuous and discrete phases, respectively; m, state of minimum fluidization; upper prime, pulsation quantities, as well as differentiation of $K_j$ with respect to $\rho$; upper asterisk, dimensionless quantities.

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